Lecture 21: Beyond One Speed (Two-Group Diffusion Theory)

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

March 2, 2026

Introduction: The Limit of "One-Speed"

Until now, we have assumed all neutrons behave like a single "gas" diffusing through the core. We lumped everything into effective constants (D, Σ_a).
The Reality:

1. Neutrons are born **Fast** (2 MeV).
2. They must slow down to become **Thermal** (0.025 eV).
3. Fast neutrons travel much further (longer Mean Free Path) than Thermal neutrons.
4. Materials like water reflect Fast neutrons differently than Thermal neutrons.

To model a real reactor with varying composition ($k_\infty(r)$), we must track these two populations separately. This is the **Two-Group Approximation**.

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1 The Two "Buckets" of Neutrons

We divide the neutron spectrum into two groups:

- **Group 1 (Fast):** Neutrons with energy $E > E_{thermal}$.
  - **Source:** Fission. (All neutrons are born here).
  - **Loss:** Scattering down to Group 2 ($\Sigma_{1\to 2}$) OR Leakage. (Absorption is usually small compared to scattering, except for resonance capture).
- **Group 2 (Thermal):** Neutrons with energy $E \approx k_B T$.
  - **Source:** Scattering down from Group 1.
  - **Loss:** Absorption (Σ_{a2}) causing fission or capture (only a fraction of absorptions cause fission).

2 The Coupled Equations

Instead of one diffusion equation ($D\nabla^2\phi + \cdots = 0$), we now have a system of two coupled differential equations.

2.1 The Fast Group (Group 1)

$$\underbrace{D_1\nabla^2\phi_1}_{\text{Diffusion}} - \underbrace{\Sigma_{1\to 2}\phi_1}_{\text{Removal to Group 2}} + \underbrace{\frac{1}{k}\nu\Sigma_{f2}\phi_2}_{\text{Source from Fission}} = 0$$

Note the coupling: The source of Fast neutrons comes from Fissions caused by **Thermal** neutrons (ϕ_2). We are ignoring the additional contribution of fissions due to fast neutrons, reasonable due to the huge difference in fission capture cross-sections.

2.2 The Thermal Group (Group 2)

$$\underbrace{D_2\nabla^2\phi_2}_{\text{Diffusion}} - \underbrace{\Sigma_{a2}\phi_2}_{\text{Absorption}} + \underbrace{\Sigma_{1\to 2}\phi_1}_{\text{Source from Group 1}} = 0$$

Note the coupling: The source of Thermal neutrons is the ”slowing down” from the **Fast** group (ϕ_1).

3 Why this matters: The Reflector Hump

The power of Two-Group theory becomes visible at the edge of the core, near the reflector (water).

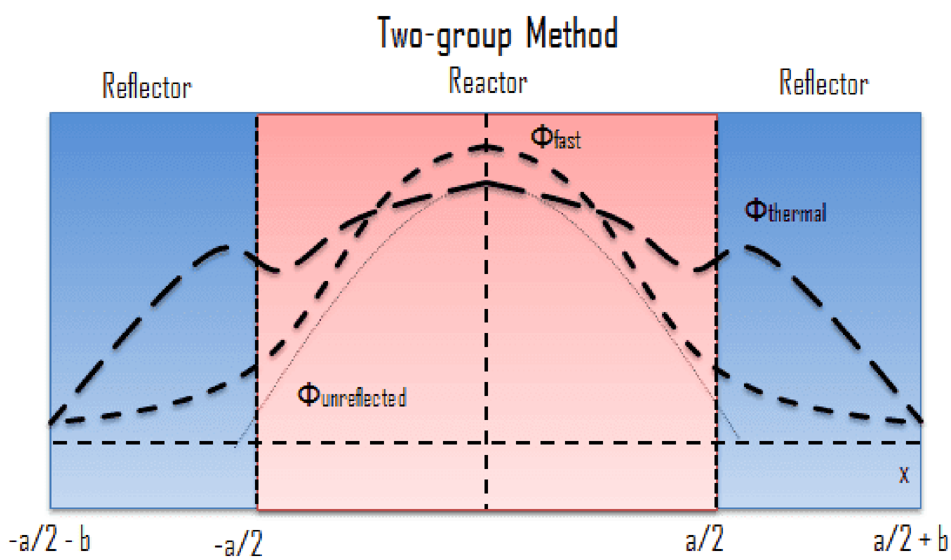

Figure 1: Two-Group neutron flux distribution in a reflected reactor. Note the rise in thermal flux (ϕ_2) in the reflector region, known as the ”Reflector Hump,” caused by the slowing down of fast leakage neutrons.

1. **Core:** Both Fast (ϕ_1) and Thermal (ϕ_2) fluxes are high (Cosine shape).
2. **Interface:** Fast neutrons leak out of the core into the water.
3. **Reflector (Water):**
   - There is no fission, so Fast neutrons are just lost or slowed down.

- The water is excellent at slowing neutrons down. The "Removal" from Group 1 becomes the "Source" for Group 2.
- **The Result:** The Thermal Flux (ϕ_2) actually **rises** in the reflector near the wall!

This **"Reflector Hump"** means there is a surplus of thermal neutrons right at the edge of the core, which can diffuse *back in* to cause more fissions at the periphery. This explains "Reflector Savings" physically—the reflector acts as a "Thermal Neutron Factory" powered by the leaking "Fast Neutron Waste."

4 The Six-Factor Formula

With Two-Group theory, our Criticality Condition ($k_{eff} = 1$) becomes more sophisticated. We replace the 4-Factor Formula with the **6-Factor Formula**:

$$k_{eff} = \eta f p \epsilon P_{FNL} P_{TNL}$$

Where we explicitly separate leakage:

- P_{FNL}: Fast Non-Leakage Probability. (Likelihood a fast neutron slows down before leaking).
- P_{TNL}: Thermal Non-Leakage Probability. (Likelihood a thermal neutron is absorbed before leaking).

In a water reactor (LWR):

- Fast neutrons travel far (τ is large). Leakage is dominated by P_{FNL}.
- Thermal neutrons don't travel far (L^2 is small). $P_{TNL} \approx 1$.

Two-group theory tells us: Leakage is a Fast Neutron problem.

5 Computational Reality

While we solve 2-group problems by hand for simple slabs, real industry calculations use computer codes (like MCNP or CASMO/SIMULATE) that use:

- **Groups:** 2 (LWR global), 7 (CANDU), or even thousands (Spectrum codes).
- **Mesh:** The reactor is divided into thousands of nodes (x, y, z).
- **Iteration:** The computer guesses a flux profile, calculates the k_∞ distribution, solves the matrix, and repeats until convergence.

This allows us to model the exact scenario you asked about: a reactor where control rods are halfway in, enrichment varies, and poisons are burning out non-uniformly.

References

1. **Textbook:** Lamarsh, J.R. and Baratta, A.J., *Introduction to Nuclear Engineering*, 4th Edition. Section 6.7 (Multigroup Calculations).
2. **Visualization:** Wikipedia, *Neutron Reflector*. https://en.wikipedia.org/wiki/Neutron_reflector